Thus, when you change coordinates, the Jacobian-type factors come from the \( \frac{\partial}{\partial x^\mu} \) and \( dx^\nu \) terms.

The components of the tensor itself do not transform, rather, the basis elements of \( V^\mu \) and \( V_\nu^* \) transform.

Thus, the tensor \( T' \) does, as defined above, transform like a tensor. The reason why this isn't our favourite covariant derivative is that had we defined \( \nabla \) with respect to a different chart we would get different components when we transform it to our original chart. This derivative operator is basis dependent.